

THE PREDICTION OF HYDRODYNAMIC VISCOUS DAMPING OF
OFFSHORE STRUCTURES IN WAVES AND CURRENT BY
THE STOCHASTIC LINEARIZATION METHOD

By

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ABSTRACT

The hydrodynamic viscous damping and response statistics of certain kinds of offshore structures subjected to unidirectional wave-current excitation have been predicted. By applying the technique of stochastic linearization, the nonlinear equation for dynamic response is linearized and then approximate predictions are made for the damping and response. Examination of the final derived relations reveals that both waves and current, in addition to contributing to the excitation force, cause additional damping. Usefulness of the stochastic linearization technique has been demonstrated by analyzing a shallow water Caisson and a deep water guyed tower.

In addition to the dynamic response, the equation for the mean static response under the combined wave-current excitation has been derived. It has been demonstrated that the mean static response depends on the seastate in addition to current.

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1.0 INTRODUCTION

1.1 General Remarks

Efforts to make more accurate prediction of response of structures in random environment have resulted in increased application of probabilistic methods. The theory for prediction of stationary response of linear structures to Gaussian excitation is well developed. However, many structures, including offshore structures, exhibit different forms of nonlinearity. When non-linearities are important the response probability distributions are no longer Gaussian and in general closed form solutions do not exist.

Over the last two decades different approximate techniques have been developed for solution of this problem. However, their application to offshore structures is extremely limited.

1.2 Purpose and Scope of the Present Study

In the present study, a method of prediction of the wave-current induced response of offshore structures is presented using the Stochastic Linearization method. Because of the inclusion of ocean current, the flow velocity is a nonzero-mean Gaussian random process. This causes the nonlinear hydrodynamic force to be asymmetric. It will be shown that despite the asymmetry the Stochastic Linearization method yields equations of manageable complexity.

Two structures of completely different dynamic characteristics have been chosen as examples. One is a stiff, shallow water single pile caisson; and the other is a compliant, deep water fixed tower. The equivalent linear viscous damping

and the response quantities are predicted in the presence of waves and current. A number of parametric studies have been conducted to demonstrate the influence of different structural parameters on the dynamic behaviour.

The main focus of this study is on the prediction of response and viscous hydrodynamic damping in moderate sea states.

2.0 NONLINEAR RANDOM VIBRATION

Strictly speaking, linearity in structures is never completely realized. Nonlinearities are present in stiffness, damping, mass, or in any combination of these. However, for offshore structures a primary source of nonlinearity is the nonlinear hydrodynamic force.

There are several approximate methods for dealing with nonlinear random vibration problems. The particular problem of an oscillator with nonlinear stiffness and excited by white noise is the only problem that has been solved exactly. The method of solution is commonly known as the Markov Vector method (Lin,1968). The differential equation for the joint probability density of response displacement and velocity, called the Fokker-Planck equation, has been set up and solved exactly. However, for nonlinear damping, the Fokker-Planck equation can be solved numerically only. Recently, this method has been applied to offshore structures (Rajagopalon, 1982; Brouwers,1982).

In the perturbation method, the solution is expanded in a power series. First developed by Crandall (Crandall,1963), this approach has later been generalized to multiple degree of freedom systems.

Generally, the solution is approximated by the first few terms of the series only. The higher order terms, particularly for nonwhite excitation, is very laborious to obtain. The response is approximated as a Gaussian process. Offshore steel jacket platforms have been analysed by this method (Taylor, 1982).

The functional power series expansion, suggested by Volterra (Volterra, 1930), has been used by Weiner (Weiner, 1958) for the nonlinear stochastic process.

In the Gaussian Closure technique, relations between second and higher order moment statistics are derived from the equation of motion. The higher order moments are decomposed into the lower order moments using special Gaussian properties. Finally, application of Fourier Transformation yields relations between cross and auto spectra involving the response and excitation quantities. This technique was proposed by Iyenger (Iyenger, 1975) and later applied to offshore structures by Dunwoody (Dunwoody, 1980).

3.0 STOCHASTIC LINEARIZATION METHOD

Introduced by Booton (Booton, 1954), this technique has been generalised by Foster (Foster, 1968) and later by Iwan and Yang (Iwan, 1972). This method can handle nonlinearities in stiffness as well as damping. The reason for choosing for this particular method over others is that it is easier to use. Also the reliability of this method is already proven.

The main steps in application of this technique are as follows:

- (1) Replace nonlinear terms with equivalent linear terms.
- (2) Find the error, which is the difference between the approximate linear and the nonlinear terms.
- (3) Take the square of error and evaluate its expected value or first moment.
- (4) Minimize the mean square error by taking its derivative with respect to the equivalent linear coefficients.
- (5) Equate these derivatives to zero to obtain the relations between linear coefficients and known problem parameters.

Consider the general equation of motion of a dynamic system

$$\ddot{x} + g(x, \dot{x}) = f(t) \quad (3.1)$$

where $g(x, \dot{x})$ is a nonlinear function of x and \dot{x} but with the limitation that it is single valued and not an even function. Assume that an approximate solution is obtained from the linear equation

$$\ddot{x} + \beta_e \dot{x} + k_e x = f(t) \quad (3.2)$$

The error introduced by linearization is:

$$\epsilon = g(x, \dot{x}) - \beta_e \dot{x} - k_e x \quad (3.3)$$

Where ϵ is a random variable.

The error is minimized by requiring the mean square error to be minimum, which implies

$$\frac{\partial}{\partial \dot{x}_e} \langle \dot{x}^2 \rangle = 0 \quad (3.4)$$

$$\frac{\partial}{\partial x_e} \langle \dot{x}^2 \rangle = 0 \quad (3.5)$$

Substituting Eq. (3.3) into Eqs. (3.4) and (3.5) and interchanging the order of differentiation and expectation we obtain from Eq. (3.4)

$$\left\langle \frac{\partial}{\partial \beta_e} [g(x, \dot{x}) - \beta_e \dot{x} - k_e x]^2 \right\rangle = 0$$

i.e.,

$$\langle g(x, \dot{x}) \dot{x} - \beta_e \dot{x}^2 - k_e x \dot{x} \rangle = 0 \quad (3.6)$$

Similarly from Eq. (3.5)

$$\langle g(x, \dot{x}) x - \beta_e x \dot{x} - k_e x^2 \rangle = 0 \quad (3.7)$$

Since the joint expectation of a random variable and its derivative is zero, Eqs. (3.6) and (3.7) lead to

$$\beta_e = \frac{\langle g(x, \dot{x}) \dot{x} \rangle}{\langle \dot{x}^2 \rangle} \quad (3.8)$$

$$k_e = \frac{\langle g(x, \dot{x}) x \rangle}{\langle \dot{x}^2 \rangle} \quad (3.9)$$

If we further restrict ourselves to the case where the stiffness and damping nonlinearities are separable then we can write

$$g(x, \dot{x}) = f_1(x) + f_2(\dot{x}) \quad (3.10)$$

where $f_1(x)$ and $f_2(\dot{x})$ are nonlinear functions of x and \dot{x} respectively.

Substitution of Eq. (3.10) in Eqs. (3.8) and (3.9) results in

$$\beta_e = \frac{\langle \dot{x} f_2(\dot{x}) \rangle}{\langle \dot{x}^2 \rangle} \quad (3.11)$$

$$k_e = \frac{\langle x f_1(x) \rangle}{\langle x^2 \rangle} \quad (3.12)$$

From Eqs. (3.11) and (3.12) we observe that the equivalent linear coefficients are obtained by equating the input-output cross correlations of the nonlinearities (e.g., $\dot{x} f_2(\dot{x})$) and their linear approximations (e.g., $\dot{x} \beta_e \dot{x}$).

4.0 DAMPING AND RESPONSE TRANSDUCER IN WAVES AND CURRENT

4.1 Formulation

Using a beam model, the flexural vibration equation of an offshore structure (e.g. Caisson, Jacket or Guyed Tower) can be written as

$$m_s(x) \frac{\partial^2 Y}{\partial t^2} + EI \frac{\partial^4 Y}{\partial x^4} + C_s \frac{\partial Y}{\partial t} + M \frac{\partial^2 Y}{\partial t^2} \delta(x-l) = f(x,t) \quad (4.1)$$

Where: $m_s(x)$ = Steel mass/unit length
 EI = Flexural rigidity
 C_s = Structural damping coefficient
 M = Deck mass

$f(x,t)$ is the loading due to waves and current, which according to Morison Formula is given by

$$f(x,t) = C_M \rho V \ddot{u} - (C_M - 1) \rho V \ddot{Y} + \frac{1}{2} \rho C_D D (u + V - \dot{Y}) |u + V - \dot{Y}| \quad (4.2)$$

Where: $V(x)$ = Volume of structure/unit length
 C_M = Added mass coefficient
 ρ = Mass density of water
 $u(x,t)$ = Wave induced horizontal water particle velocity
 $V(x)$ = Current speed
 D = Total projected area perpendicular to the flow

The formulation is based on the assumption that the Morison Formula can be applied to express drag in terms of the relative fluid-structure velocity.

The first two terms of $t(x,t)$ stand for the inertia force and the third term stands for the drag force.

The hydrodynamic load model may not be very accurate for multiple leg offshore structures like a Jacket or a Guyed Tower. This is because the spatial effect on the phase of wave forces on legs and the loading on secondary cross members are neglected.

Although the structural idealisation is crude, its consequences are not serious. The simple beam model idealisation adopted here is not used for solving the Eigenvalue problem. In the context of the present work, the Eigenvalue solution is assumed to be known. The structural model adopted here is satisfactory for investigation of the nonlinear effects of seastate and current on dynamic response behaviour, and viscous hydrodynamic damping.

4.1.1 Linearization

The presence of current causes asymmetric drag force. Since the drag force is a random process with a non-zero mean, the stationary solution of Eq. (4.1) can be written in the following form as a superposition:

$$y(x,t) = y_0(x) + \gamma(x,t) \quad (4.3)$$

where $y_0(x)$ = Mean deterministic offset

$\gamma(x,t)$ = Time dependent response

Substituting Eqs. (4.2) and (4.3) into Eq. (4.1) we obtain

$$m_s \frac{\partial^2 \gamma}{\partial t^2} + EI \frac{\partial^4 \gamma}{\partial x^4} + EI \frac{\partial^4 y_0}{\partial x^4} + C_s \frac{\partial \gamma}{\partial t} + M \frac{\partial^2 \gamma}{\partial t^2} \delta(x-l)$$

$$= C_M \rho V \dot{u} - (C_M - 1) \rho V \dot{\gamma} + \frac{1}{2} \rho C_D D (u + V - \dot{\gamma}) |u + V - \dot{\gamma}|$$

This equation can be recast in the following form

$$m_s \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} + C_s \frac{\partial y}{\partial t} + M \frac{\partial^2 y}{\partial t^2} \delta(x-1) = C_M \rho \nabla \dot{u} - (C_M - 1) \rho \nabla \dot{y} + g(r) \quad (4.4)$$

Where

$$g(r) = \frac{1}{2} \rho C_D D(r+V) |r+V| - EI \frac{\partial^4 y_0}{\partial x^4} \quad (4.5)$$

$g(r)$ is a nonlinear function of r , which is the fluid-structure relative velocity, defined as

$$r = u - \dot{y} \quad (4.6)$$

In the stochastic linearization approach, $g(r)$ is replaced by $C_e r$, where C_e is the equivalent linear coefficient. The meansquare error between the nonlinear term and its linear approximation is given by

$$\langle [g(r) - C_e r]^2 \rangle = \varepsilon \quad (4.7)$$

The equivalent linear coefficient is obtained by equating the derivative to zero.

$$\frac{\partial \varepsilon}{\partial C_e} = \frac{\partial}{\partial C_e} \langle [g(r) - C_e r]^2 \rangle \quad (4.8)$$

Interchanging the order of differentiation and expectation leads to

$$\langle 2[g(r) - C_e r]r \rangle = 0$$

$$C_e = \frac{\langle g(r)r \rangle}{\langle r^2 \rangle} \quad (4.9)$$

The quantity $\langle g(r)r \rangle$ is very difficult to evaluate, because $g(r)$ is related to r in a very complex manner. However, the quantity on the right hand side of Eq. (4.9) satisfies the following Gaussian identity.

$$\frac{\langle g(r)r \rangle}{\langle r^2 \rangle} = \langle \frac{d}{dr} g(r) \rangle \quad (4.10)$$

Proof of this identity is in Appendix A.

It turns out that for $g(r)$ as defined in Eq. (4.5), the right hand side quantity of Eq. (4.10) is far simpler to evaluate than the left hand side quantity.

The equivalent linear coefficient is

$$C_e = \langle \frac{d}{dr} g(r) \rangle \quad (4.11)$$

The equivalent linear coefficient, as defined by Eq. (4.11), is the expected value of the gradient of the nonlinear function $g(r)$.

Substituting Eq. (4.5) into Eq. (4.11) and carrying out the differentiation we get

$$C_e = \frac{1}{2} \rho C_D \langle 2|r+V| \rangle \quad (4.12)$$

$(r+V)$ is a non zero mean Gaussian random variable. The quantity $|r+V|$ is evaluated in Appendix B.

Using the result of Appendix B

$$C_e = \frac{1}{2} \rho C_D D \left[\sqrt{\frac{8}{\pi}} \text{ or } \exp\left(-\frac{V^2}{2\sigma r^2}\right) + 2V \text{Erf}\left(\frac{V}{\sqrt{2}\sigma r}\right) \right] \quad (4.13)$$

Where Erf(x) is the Error Function defined as

$$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$$

It is also known as the Probability Integral.

or is the rms fluid-structure relative velocity.

Spanos investigated the case of a single degree-of-freedom system under wave-current loading and arrived at similar expression for the equivalent damping coefficient (Spanos, 1981).

Replacing the nonlinear term $g(r)$ with the equivalent linear term, Eq. (4.4) can recast in the following form

$$\left[m_s + (C_M - 1) \rho V \right] \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} + (C_s + C_e) \frac{\partial y}{\partial t} + M \frac{\partial^2 y}{\partial t^2} \delta(x-L) \quad (4.14)$$

$$= C_M \rho V \dot{u} + C_e u$$

From Eqs. (4.13) and (4.14) we observe that waves and current influence excitation on the structure as well as damping of the structure.

A component of damping is a function of sea state and current. Similarly there is a sea state and current dependent exciting force.

4.1.2 Solution

The structure response is expressed in terms of its normal modes ϕ_i and generalised coordinates q_i .

$$y(x,t) = \sum_{i=1}^N \phi_i(x) q_i(t) \quad (4.15)$$

For the i th mode, the uncoupled modal equation is

$$M_i \ddot{q}_i + C_i \dot{q}_i + K_i q_i = N_i \quad (4.16)$$

Where: M_i = Modal mass =

$$\int_0^L m_s(x) \phi_i^2 dx + (C_M - 1) \rho \int_0^d \nabla \phi_i^2 dx + M \phi_i^2 \delta(x-L)$$

K_i = Modal stiffness =

$$\omega_i^2 M_i$$

$$C_i = \text{Modal damping} = \int_0^L C_s \phi_i^2 dx + \int_0^d C_e \phi_i^2 dx$$

$$= 2M_i \omega_i \xi_{si} + \frac{1}{2} \rho C_D D \sqrt{\frac{8}{\pi}} \int_0^d D \exp(-V^2/2\sigma r^2) \sigma r \phi_i^2 dx$$

$$+ \frac{1}{2} \rho C_D \int_0^d 2DV \operatorname{Erf}(V/\sqrt{2}\sigma r) \phi_i^2 dx$$

ξ_{si} = Modal non viscous damping ratio

d = Depth of water

L = Length of structure

Therefore the modal damping ratio

$$\xi_i = \frac{C_i}{2M_i \omega_i} = \xi_{si} + \xi_{vi} \quad (4.18)$$

Where ξ_{vi} = Modal viscous damping ratio

$$\begin{aligned} &= \frac{C_D}{4M_i \omega_i} \left[\sqrt{\frac{8}{\pi}} \int_0^d D \exp(-V^2/2\sigma r^2) \sigma r \phi_i^2 dx \right. \\ &\quad \left. + \int_0^d 2DV \operatorname{Erf}\left(\frac{V}{\sqrt{2}\sigma r}\right) \phi_i^2 dx \right] \end{aligned} \quad (4.19)$$

The modal wave-current induced exciting force is given by

$$\begin{aligned} N_i &= C_M \rho \dot{u}_0 \int_0^d \nabla Z \phi_i dx + \frac{\rho C_D}{2} U_0 \left[\sqrt{\frac{8}{\pi}} \int_0^d D \exp(-V^2/2\sigma r^2) \sigma r \phi_i z dx \right. \\ &\quad \left. + \int_0^d 2DV \operatorname{Erf}(V/\sqrt{2}\sigma r) Z \phi_i dx \right] \end{aligned} \quad (4.20)$$

In Eq. (4.20) the following relations have been used for the wave induced water particle velocity and acceleration at any arbitrary depth.

$$u = u_0 Z \quad (4.21)$$

$$\dot{u} = \dot{u}_0 Z$$

Where: u_o = Wave induced water particle velocity on the
free surface

\dot{u}_o = Wave induced water particle acceleration on the
free surface

z = Depth attenuation factor = $\frac{\cosh(kx)}{\sinh(kd)}$

k is the wave number.

In terms of the modal quantities, the variance of the structural response is

$$\sigma_s^2 = \left\langle \left(\sum_{i=1}^N \phi_i q_i \right)^2 \right\rangle \quad (4.22)$$

For Caisson or Jacket type offshore structures, the natural frequencies are widely separated and the damping is light. Because of this, the responses in different modes can be considered to be stochastically uncorrelated. With this widely used established hypothesis in structural dynamics, the cross terms on the right hand side of Eq. (4.22) vanish and it simplifies to

$$\sigma_s^2 = \left\langle \sum_{i=1}^N \phi_i^2 q_i^2 \right\rangle = \sum_{i=1}^N \phi_i^2 \sigma_{q_i}^2 \quad (4.23)$$

Where: σ_{q_i} = rms modal response in the i th. mode
 σ_s = rms response of structure

A simple expression for the variance of the relative fluid-structure velocity can be derived, if the cross correlation between the fluid particle velocity and the structural velocity is neglected.

$$\sigma_r^2 = \langle (u - \dot{y})^2 \rangle = \langle u^2 \rangle + \langle \dot{y}^2 \rangle = \langle u_o^2 z^2 \rangle + \langle \left(\sum_{i=1}^N \phi_i \dot{q}_i \right)^2 \rangle$$

In the above expression, the cross correlation term has been neglected.

Substituting Eq. (4.23) in this equation leads to

$$\sigma_r^2 = z^2 \sigma_{u_o}^2 + \sum_{i=1}^N \phi_i^2 \sigma_{q_i}^2 \omega_i^2 \quad (4.24)$$

ω_i is the natural frequency corresponding to i th mode.

Eqs. (4.19), (4.20) and (4.24) indicate that the damping and excitation in any particular mode depend on the seastate and responses in all modes. This is a manifestation of the nonlinear drag force.

The response spectra of an offshore structure has two types of peaks. One corresponds to the quasi-static response of the structure and is located at the wave spectral peak frequency. The other type is due to the resonant dynamic response of the structure at its natural frequencies. In this study we are primarily concerned about low to moderate sea states and hence the quasi-static response is negligible. Therefore, the response variance can be computed considering the dynamic response peak only.

Since an offshore structure is a lightly damped system, the modal response variance can be estimated by the half power bandwidth method as follows

$$\sigma_{qi}^2 = \frac{1.25 S_i(\omega) \big|_{\omega=\omega_i}}{2 M_i^2 \omega_i^3 \xi_i} \quad (4.25)$$

Where: $S_i(\omega)$ = Spectral density function of modal exciting force corresponding to i th. mode

A compact expression for the modal exciting force can be written using the two auxiliary variables C_a and C_u .

$$N_i = C_a \dot{u}_o + C_u u_o \quad (4.26)$$

Since the fluid particle velocity and acceleration are stochastically uncorrelated, the spectral density function of modal exciting force is

$$S_i(\omega) = C_a^2 S_{\dot{u}_o}(\omega) + C_u^2 S_{u_o}(\omega) \quad (4.27)$$

Where: $S_{u_o}(\omega)$ = s.d.f of fluid particle velocity
 $= \omega^2 S_\eta(\omega)$

$S_{\dot{u}_o}(\omega)$ = s.d.f of fluid particle acceleration
 $= \omega^4 S_\eta(\omega)$

$S_\eta(\omega)$ is the s.d.f of wave elevation.

Substituting in Eq. (4.27) yields

$$S_i(\omega) = S_\eta(\omega) (C_a^2 \omega^4 + C_u^2 \omega^2) \quad (4.28)$$

Where:

$$C_u = \frac{\rho C_D}{2} \left[\frac{\pi}{2} \int_0^d D \exp(-V^2/2\sigma r^2) \sigma r \phi_i Z dx \right. \\ \left. + \int_0^d 2DV \operatorname{Erf}(V/\sqrt{2}\sigma r) \phi_i Z dx \right]$$

$$C_a = C_M \rho \int_0^d \nabla z \phi_i dx$$

The mean response of the structure can be determined by requiring that the solution given by Eq. (4.3) satisfies Eq. (4.4) on the average.

This requirement leads to the following equation

$$\langle g(r) \rangle = 0$$

$$EI \frac{d^4 y_0}{dx^4} = \frac{1}{2} \rho C_D \langle (r+V) | r+V | \rangle \quad (4.29)$$

The expected value of the quantity on the right hand side of Eq. (4.29) has been derived in Appendix C.

Substituting the result of Appendix C in Eq. (4.29) leads to

$$EI \frac{d^4 y_0}{dx^4} = \frac{1}{2} \rho C_D D [(V^2 + \sigma r^2) \text{Erf}(V/\sqrt{2}\sigma r) + \sqrt{\frac{2}{\pi}} V \sigma r \exp(-V^2/2\sigma r^2)] \quad (4.30)$$

This equation is the well known beam equation and the right hand side quantity is the mean, time independent wave-current induced load. It is interesting to note that the mean load is a function of the sea state in addition to current. The dependence on the sea state is caused by the nonlinearity in drag force. If the drag force was linear, the mean load would have been a function of current only.

The mean structural deflection can be obtained by repeated integration of Eq. (4.30) and application of appropriate boundary conditions.

4.2 Computation

It is apparent from Eqs. (4.19) and (4.20) that the modal viscous damping and exciting force depend on the structural response. This necessitates the adoption of an iterative procedure for the determination of the damping and response. From the computation done in this study, it has been observed that the rate of convergence depends primarily on the response of the structure. When the structural response is small in comparison to fluid particle motion, the rate of convergence is rapid.

The modal quantities given by Eqs. (4.19) and (4.20) are evaluated by numerical integration, because most of the quantities in the integrands are not defined analytically. For numerical integration, Simpson three point rule is used. In the software developed, five equally spaced integration stations are used over the length of the structure. The numerical integration has the advantage that any arbitrary current profile and mode shape can be handled. These quantities need to be defined numerically at the discrete integration stations.

The damping and response prediction is made using the two parameter Bretschneider wave spectrum defined as

$$S_{\eta}(\omega) = .3125 H_s^2 \frac{\omega_p^4}{\omega^5} \exp\left(-1.25 \frac{\omega_p^4}{\omega^4}\right)$$

Where: H_s = Significant wave height (ft.)
 ω_p = Wave frequency corresponding to spectral peak (rad/sec)

The peak frequency (ω_p), if not known, is estimated in the following way. For Bretschneider spectra, the peak frequency is evaluated using the following relation between the peak frequency and the average zero crossing period (T_z) (Sarpkaya, 1981).

$$\omega_p = 4.4588/T_z$$

The relation between the significant wave height (H_s) and the average zero crossing period is probabilistic in nature. However, for simplicity the following deterministic relation, proposed by Weigel (Weigel, 1978), has been used.

$$T_z = 1.723 H_s^{.56} \quad (4.32)$$

T_z is in sec. and H_s is in ft.

The Error Function in Eqs. (4.19) and (4.20) is approximated by the following (Abramowitz and Stegun, 1965)

$$\text{Erf}(x) = 1 - \frac{1}{(1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)^4} \quad (4.33)$$

Where: $a_1 = .278393$
 $a_2 = .230389$
 $a_3 = .000972$
 $a_4 = .078108$

Some other approximations were tried. But this particular one has been found to be well behaved and works better than some others. This approximation is always within the bounds of the Error Function i.e., between zero and one.

4.2.1 The Caisson With Low Response Amplitude

A closed form expression can be derived for the modal viscous damping when the structural response is negligible compared to water particle motion. This is done for the first mode of a caisson in this section.

Since the structural motion is negligible, Eq. (4.24) becomes

$$\sigma_r = \sigma_{uo} Z \quad (4.34)$$

Where Z is the Depth Attenuation Factor. It is assumed that current V obeys the same Depth Attenuation law. This assumption makes the derivation considerably simpler. The choice of current distribution profile is not crucial. Since, it has been shown in the later part of this work that current distribution has negligible influence on the damping and dynamic response.

Based on the assumption made, the current speed at any depth can be expressed as

$$V = V_o Z$$

Where: V_o = Current speed at free surface

Substituting Eqs. (4.34) and (4.35) in Eq. (4.19) yields

$$\begin{aligned} \xi_{Vi} = & \frac{\rho C_D D}{4M_1 \omega_1} \left[\sqrt{\frac{8}{\pi}} \int_0^d \exp(-V_o^2 / 2\sigma_{uo}^2) \sigma_{uo} Z \phi_i^2 dx \right. \\ & \left. + \int_0^d 2V_o Z \operatorname{Erf}(V_o / \sqrt{2}\sigma_{uo}) \phi_i^2 dx \right] \end{aligned} \quad (4.36)$$

$$\begin{aligned} = & \frac{\rho C_D D}{4M_1 \omega_1} \left[\sqrt{\frac{8}{\pi}} \exp(-V_o^2 / 2\sigma_{uo}^2) \sigma_{uo} + 2V_o \operatorname{Erf}(V_o / \sqrt{2}\sigma_{uo}) \right] \\ & \times \int_0^d Z \phi_i^2 dx \end{aligned}$$

The first mode of a caisson can be approximated by the following trigonometric function

$$\phi_1(x) = 1 - \cos\left(\frac{\pi x}{2L}\right) \quad (4.37)$$

Substituting this approximation for the mode shape, the integration in Eq. (4.36) can be evaluated as follows.

Calling this integration $G(\phi)$, we can write

$$\begin{aligned} G(\phi) &= \int_0^d z \phi_1^2 dx = \int_0^d e^{k(x-d)} (1 - \cos \frac{\pi x}{2L})^2 dx \\ &= e^{-kd} \int_0^d e^{kx} (1 - \frac{1}{2} (\exp(\frac{i\pi x}{2L}) + \exp(-\frac{i\pi x}{2L})))^2 dx \\ &= e^{-kd} \int_0^d e^{kx} (\frac{3}{2} + \frac{1}{4} \exp(\frac{i\pi x}{L}) + \frac{1}{4} \exp(-\frac{i\pi x}{L}) \\ &\quad - \exp(\frac{i\pi x}{2L}) - \exp(-\frac{i\pi x}{2L})) dx \end{aligned}$$

Performing termwise integration leads to

$$\begin{aligned} G(\phi) &= e^{-kd} \left[\frac{3}{2k} (e^{kd} - 1) + \frac{1}{4} \frac{e^{kd} (2kL \cos \frac{\pi d}{L} + 2 \sin \frac{\pi d}{L})}{(k^2 L^2 + \pi^2)} \right. \\ &\quad - \frac{2L e^{kd}}{4k^2 L^2 + \pi^2} (4kL \cos \frac{\pi d}{2L} + 2 \sin \frac{\pi d}{2L}) \\ &\quad \left. - \frac{kL^2}{2(k^2 L^2 + \pi^2)} + \frac{8kL^2}{4k^2 L^2 + \pi^2} \right] \end{aligned}$$

Since $kd \gg 1$ $e^{kd} - 1 = e^{kd}$

$$\begin{aligned}
 G(\phi) = & \frac{3}{2k} + \frac{L}{4(k^2 L^2 + \pi^2)} (2kL \cos \frac{\pi d}{L} + 2\pi \sin \frac{\pi d}{L}) \\
 & - \frac{2L}{4k^2 L^2 + \pi^2} (4kL \cos \frac{\pi d}{2L} + 2 \sin \frac{\pi d}{2L}) \\
 & - \frac{kL^2}{2e^{kd} (k^2 L^2 + \pi^2)} + \frac{8kL^2}{e^{kd} (4k^2 L^2 + \pi^2)}
 \end{aligned} \tag{4.38}$$

Further simplification can be achieved when the depth of water is nearly equal to caisson length, i.e. $d=L$.

Under this condition

$$\begin{aligned}
 G(\phi) = & \frac{3}{2k} - \frac{kL^2}{2(k^2 L^2 + \pi^2)} - \frac{4L\pi}{4k^2 L^2 + \pi^2} \\
 & - \frac{kL^2}{2e^{kd} (k^2 L^2 + \pi^2)} + \frac{8kL^2}{e^{kd} (4k^2 L^2 + \pi^2)}
 \end{aligned} \tag{4.39}$$

The r.m.s fluid particle velocity

$$\sigma_{uo} = \frac{2\pi}{T_z} \frac{H_s}{4} \tag{4.40}$$

It can be expressed as a function of significant wave height only by substituting Eq. (4.32) in Eq. (4.40).

Then Eq.(4.40) becomes

$$\sigma_{uo} = \frac{2\pi}{1.723 H_s^{.56}} \times \frac{H_s}{4} = .91 H_s^{.44} \tag{4.41}$$

Furthermore,

$$\frac{V_o}{\sqrt{2}\sigma_{uo}} = \frac{.78V_o}{H_s^{.44}} \quad (4.42)$$

and

$$\frac{V_o^2}{2\sigma_{uo}^2} = \frac{.61V_o^2}{H_s^{.88}} \quad (4.43)$$

For the wave number (k), using the deep water dispersion relation and Eq. (4.32) leads to

$$k = \frac{\omega^2}{g} = \left(\frac{2\pi}{T_z}\right)^2 \frac{1}{g} = \frac{4\pi^2}{g \times (1.723H_s^{.56})^2} = \frac{.412}{H_s^{1.12}} \quad (4.44)$$

Substituting Eqs. (4.41), (4.42) and (4.43) in Eq. (4.36) yields

$$\begin{aligned} \xi_{Vi} &= \frac{\rho C_D D}{4M_1 \omega_1} [1.6 \times .91 H_s^{.44} \exp\left(-\frac{.61V_o^2}{H_s^{.88}}\right) + 2V_o \text{Erf}\left(\frac{.78V_o}{H_s^{.44}}\right)] G(\phi) \\ &= \frac{\rho C_D D}{4M_1 \omega_1} [1.456 H_s^{.44} \exp\left(-\frac{.61V_o^2}{H_s^{.88}}\right) + 2V_o \text{Erf}\left(\frac{.78V_o}{H_s^{.44}}\right)] G(\phi) \end{aligned} \quad (4.45)$$

$G(\phi)$, which is a function of the mode shape, is given by either Eq. (4.38) or Eq. (4.39)

The wave number (k), which is needed in the evaluation of $G(\phi)$, is given by Eq. (4.44)

Eq. (4.45) provides a means for the prediction of modal viscous damping of a caisson in presence of waves and current. This equation is not very accurate for the prediction purposes, but it gives a feel for relative influences of different environmental parameters on the damping.

4.2.2 Flexible Caisson

The Caisson analyzed in this section is located at West Cameron Block 32 in the Gulf of Mexico. It is owned and operated by Mobil Oil Co. The particulars of the Caisson are as below.

Total Length	= 230.0 ft.
Water Depth	= 175.0 ft.
Leg Diameter	= Tapers From 16 ft. at Mudline to 8 ft. at the Waterline
Steel Mass	= Varies From 181.42 slugs/ft. at Mudline to 24.65 slugs/ft. at deck level
Deck Mass	= 2484.50 slugs
Natural Frequency	= 2.89 rad/sec.
Non viscous Damping	= 1.4%

The Caisson is assumed to respond in the first mode only and the mode shape is shown in Fig.2.

A sensitivity analysis has been carried out and the influence of the following engineering parameters on the dynamic behaviour of the Caisson has been studied.

- (1) Sea state, defined by significant wave height(H_s)
- (2) Current speed (V)
- (3) Frequency ratio, defined as the ratio of the first natural frequency of the structure (ω_n) and the peak spectral frequency(ω_p)
- (4) Drag coefficient(C_D)
- (5) Current distribution parameter (C_v). This is defined as the ratio of the area under the current distribution profile and the area of the largest rectangle enclosing it. A linear profile has a ratio of 1/2.

Figs. 4 and 5 show the variation of damping and response with sea state and current speed. Damping increases considerably with current speed and significant wave height. The response increases with sea state (Fig. 5), but is hardly influenced by

current. It has been already demonstrated that waves and current contribute to both excitation and damping. Therefore the variation of response with changing sea state or current depends on the relative magnitudes of modifications in damping and excitation. For the Caisson, the response increases with sea state, because the increase in excitation is not fully offset by the increase in damping. Similarly, the response increases with current, because the increase in excitation is not fully offset by the increase in damping due to current.

The deck response standard deviation and the modal damping ratio are plotted against the frequency ratio (ω_n/ω_p) in Figs. 6 and 7. Damping is found to be inversely related to frequency ratio. The plot of response standard deviation vs. frequency ratio resembles the response behaviour of a deterministic dynamic system under harmonic excitation. Like the resonance of a deterministic system, the response standard deviation reaches its peak when the frequency ratio is close to one.

In the present analysis a constant drag coefficient has been used. However, the drag coefficient is a function of Reynold No. and Keulegan Carpenter No. This means that the drag coefficient is time dependent and also varies along the length of the structure. To find out the consequences of applying a constant drag coefficient, damping and response standard deviation have been plotted against the drag coefficient (C_D) in Figs. 8, 9, 10 and 11. The response is found to be rather insensitive to drag coefficient. The damping increases with the drag coefficient. From Fig. 12 it can be concluded that small variations in current

distribution have negligible effects on damping and response.

4.2.3 Guyed Tower

In this section, the results of an analysis for a Guyed Tower are presented. The purpose here is not to predict the design values, but to present characteristic response quantities. The relative significance of some important environmental and structural parameters is investigated and some specific conclusions drawn.

A Guyed Tower is a compliant structure (Fig. 13). The guy lines provide the soft restoring force. The tower is essentially a truss work built between four corner legs. The tower rests on a piled foundation. The tower base can be idealised to be free to rotate, but fixed against sliding.

The particulars of the specific tower considered are given below

Total Length	= 1804.0 ft.
Water Depth	= 1689.2 ft.
Leg Diameter	= 6.56 ft.
Steel Mass	= 530.0 slugs/ft.
Deck Mass	= 374294.0 slugs
Sway Mode Natural Frequency	= .20 rad/sec.
Bending Mode Natural Frequency	= 1.4 rad/sec.
Bending Mode Non Viscous Damping	= 2.0%

The first two natural modes of vibration are shown in Fig. 14. In the present analysis it is assumed that the tower responds only in these two modes. The first mode is a rigid body

rotation and the response in this mode is determined by the tower mass and guy stiffness. This is referred to as the sway mode. The response in the second mode, called the bending mode here, is dictated by the tower mass and flexural stiffness.

The response in the sway mode is not computed here. This mode is primarily excited by the second order wave forces and wind forces, both of which are difficult to model in the frequency domain in spectral terms. The standard deviation of the sway mode response is input as a known quantity.

The two quantities that have been predicted are the viscous damping ratio and the standard deviation of deck response in the bending mode. The second response quantity is by itself not very important because the deck response is primarily caused by the sway mode. However, the deck response in the bending mode is a measure of the tower bending moment. Because the standard deviation of bending moment is directly proportional to the standard deviation of displacement response.

The influence of the following parameters on the bending mode response has been investigated.

- (1) Significant wave height (H_g)
- (2) Current speed (V)
- (3) Standard deviation of deck response in the sway mode (σ_g)

- (4) Ratio of bending mode natural frequency and frequency of spectral peak (ω_n/ω_p)
- (5) Current distribution parameter (C_V), which has already been defined
- (6) Drag coefficient (C_D)

The sway mode motion influences the damping and excitation in the bending mode through Eq. (4.24).

Examination of Figs. 15, 16, 17, 18, 19, 20 reveals that the damping in the bending mode is very sensitive to current and the sway mode response, but very little influenced by the sea state. This is because wave induced water particle velocity attenuates very rapidly in moderate sea states. It is predominant near the free surface. Therefore, the standard deviation of relative fluid-structure velocity (σ_r) and the equivalent damping coefficient are dominated by the sway mode response and ocean current. The response in the bending mode increases initially with significant wave height (Figs. 16, 18, 20) but then flattens out for high sea states. This is probably because the spectral peak frequency moves away from the bending mode natural frequency as the significant wave height goes up.

Although the standard deviation of bending mode response decreases with increasing current (Fig. 16), the r.m.s. response will not necessarily follow the same trend. Because, the r.m.s value depends on mean deflection, which increases with current (Eq. 4.30).

In Figs. 22 and 24 the bending mode response standard deviation has been plotted against the frequency ratio. The resonance like effect is noticeable here also, like the Caisson.

This becomes more pronounced as the sea state goes up.

From Fig. 25 it is observed that damping increases and response decreases as the variation of current with depth gets more uniform.

In Fig. 26 damping and response are plotted against drag coefficients. Unlike the caisson, damping increases and response decreases with increasing drag coefficients.

5.0 SUMMARY AND CONCLUSIONS

A simple approach based on the Stochastic Linearization technique has been presented for the dynamic response analysis of offshore structures, excited by waves and current. Because of the presence of current, the drag force is asymmetric. Consequently, the stationary solution for structural response consists of superposition of a time independent deterministic offset and a randomly fluctuating component. The main focus in this study has been on the randomly fluctuating component of response. But the equation for deterministic offset has been derived.

The Stochastic Linearization technique replaces the original nonlinear drag term with an equivalent (optimal) linear drag term. The equivalent linear equation has been employed with an iterative numerical scheme, to obtain an approximate solution for the standard deviation of response of the original nonlinear system.

Parametric studies have been conducted on a stiff, shallow water structure and a compliant, deep water structure. These studies have helped to gain insight into the dynamic behaviour of offshore structures. Also, based on these studies some general comments can be made. For shallow water structures, sea state appears to have the most significant influence on the dynamic response of the structure. On the other hand, for deep water compliant structures the dynamic behaviour in the bending mode depends primarily on the compliant mode response and the ocean current. Seastate is a relatively insignificant design

parameter. However, this is not necessarily true for the response in the compliant mode. For deep water compliant structures, the response in the bending mode depends heavily on the drag coefficient value. Therefore a judicious selection of drag coefficient is important.

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APPENDIX A

Proof of Gaussian Identity:

$$\frac{\langle g(r)r \rangle}{\langle r^2 \rangle} = \langle \frac{d}{dr} g(r) \rangle$$

By definition

$$\langle \frac{d}{dr} g(r) \rangle = \int_{-\infty}^{\infty} \frac{dg(r)}{dr} p(r) dr$$

where

$p(r)$ = Gaussian probability density function for the random variable "r"

$$= \frac{1}{\sqrt{2\pi}\sigma} \exp(-r^2/2\sigma^2) \quad (A.1)$$

Integrating by parts

$$\langle \frac{d}{dr} g(r) \rangle = p(r)g(r) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{dp(r)}{dr} g(r) dr$$

Taking derivative of $p(r)$ and substituting

$$\langle \frac{d}{dr} g(r) \rangle = - \int_{-\infty}^{\infty} g(r) (-r/\sigma^2) p(r) dr$$

$$= \frac{1}{\sigma^2} \int_{-\infty}^{\infty} r g(r) p(r) dr = \frac{\langle rg(r) \rangle}{\langle r^2 \rangle}$$

APPENDIX B

Evaluation of $\langle 2|r+V| \rangle$.

The expected value of a function of a random variable can be computed without first determining the complete probability structure of the function, provided the function is a Borel function. For our purposes, it suffices to know that a function with at most a finite number of discontinuities is a Borel function.

Assuming that $|r+V|$ is a function of r , which satisfies such properties, we can write

$$\begin{aligned} \langle |r+V| \rangle &= \int_{-\infty}^{\infty} |r+V| p(r) dr = \frac{1}{\sigma r \sqrt{2\pi}} \int_{-\infty}^{\infty} |r+V| \exp(-r^2/2\sigma r^2) dr \\ &= \frac{1}{\sigma r \sqrt{2\pi}} \left[-\int_{-\infty}^{-V} (r+V) \exp(-r^2/2\sigma r^2) dr + \int_{-V}^{\infty} (r+V) \exp(-r^2/2\sigma r^2) dr \right] \end{aligned}$$

(B.1)

Breaking up the limits of the second integration we have

$$\begin{aligned} \langle |r+V| \rangle &= \frac{1}{\sigma r \sqrt{2\pi}} \left[-\int_{-\infty}^{-V} \exp(-r^2/2\sigma r^2) dr - \int_{-\infty}^{-V} \exp(-r^2/2\sigma r^2) dr \right. \\ &\quad + \int_{-V}^V V \exp(-r^2/2\sigma r^2) dr + \int_{-V}^V r \exp(-r^2/2\sigma r^2) dr \\ &\quad \left. + \int_V^{\infty} V \exp(-r^2/2\sigma r^2) dr + \int_V^{\infty} r \exp(-r^2/2\sigma r^2) dr \right] \end{aligned} \quad (B.2)$$

Application of properties of even and odd function leads to:

$$\begin{aligned} \langle |r+V| \rangle &= \frac{1}{\sigma r \sqrt{2\pi}} \left[2 \int_V^{\infty} r \exp(-r^2/2\sigma r^2) dr + 2V \int_0^V \exp(-r^2/2\sigma r^2) dr \right] \\ &= \frac{1}{\sigma r \sqrt{2}} (2I_1 + 2VI_2) \end{aligned}$$

Where the auxiliary variables

$$I_1 = \int_V^\infty r \exp(-r^2/2\sigma r^2) dr \quad (B.4)$$

and

$$I_2 = \int_0^V \exp(-r^2/2\sigma r^2) dr$$

Breaking up the limits of integration for I_1 , we have

$$I_1 = \int_0^\infty r \exp(-r^2/2\sigma r^2) dr - \int_0^V r \exp(-r^2/2\sigma r^2) dr \quad (B.4)$$

For the first integration in (B.4) the following integration result is used.

$$\int_0^\infty t^{2n+1} \exp(-at^2) dt = \frac{n!}{2a^{n+1}}$$

and we have

$$\begin{aligned} I_1 &= \frac{1}{2\left(\frac{1}{2\sigma r^2}\right)} - (-\sigma r^2) \exp(-r^2/2\sigma r^2) \Big|_0^V \\ &= \sigma r^2 + \sigma r^2 [\exp(-V^2/2\sigma r^2) - 1] \\ &= \sigma r^2 \exp(-V^2/2\sigma r^2) \end{aligned} \quad (B.5)$$

I_2 is evaluated using the following result

$$\int_0^u \exp(-q^2 x^2) dx = \frac{\sqrt{\pi}}{2q} \operatorname{Erf}(qu)$$

$$\begin{aligned}
I_2 &= \frac{\sqrt{\pi}}{2/\sqrt{2}\sigma r} \operatorname{Erf}(V/\sqrt{2}\sigma r) \\
&= \frac{\sigma r \sqrt{\pi}}{\sqrt{2}} \operatorname{Erf}(V/\sqrt{2}\sigma r)
\end{aligned} \tag{B.6}$$

Substituting Eqs. (B.5) and (B.6) in Eq. (B.3)

$$\begin{aligned}
\langle |r+V| \rangle &= \frac{1}{\sigma r \sqrt{2\pi}} [2\sigma r^2 \exp(-V^2/2\sigma r^2) \\
&\quad + 2V \frac{\sigma r \sqrt{\pi}}{\sqrt{2}} \operatorname{Erf}(V/\sqrt{2}\sigma r)] \\
&= \frac{1}{2} \sqrt{\frac{8}{\pi}} \sigma r \exp(-V^2/2\sigma r^2) + V \operatorname{Erf}(V/\sqrt{2}\sigma r)
\end{aligned}$$

Finally

$$\langle 2|r+V| \rangle = \sqrt{\frac{8}{\pi}} \sigma r \exp(-V^2/2\sigma r^2) + 2 V \operatorname{Erf}(V/\sqrt{2}\sigma r)$$

APPENDIX C

Evaluation of $\langle (r+V) | r+V \rangle$

Following the same arguments as in Appendix B

$$\begin{aligned}
 \langle (r+V) | r+V \rangle &= \int_{-\infty}^{\infty} (r+V) |r+V| p(r) dr \\
 &= \frac{1}{\sigma r \sqrt{2\pi}} \left[- \int_{-\infty}^{-V} (r+V)^2 \exp(-r^2/2\sigma r^2) dr \right. \\
 &\quad \left. + \int_{-V}^{\infty} (r+V)^2 \exp(-r^2/2\sigma r^2) dr \right] \\
 &= \frac{1}{\sigma r \sqrt{2\pi}} \left[- \int_{-\infty}^{-V} V^2 \exp(-r^2/2\sigma r^2) dr - \int_{-V}^V r^2 \exp(-r^2/2\sigma r^2) dr \right. \\
 &\quad - 2V \int_{-\infty}^{-V} r \exp(-r^2/2\sigma r^2) dr + \int_{-V}^V V^2 \exp(-r^2/2\sigma r^2) dr \\
 &\quad + \int_{-V}^V r^2 \exp(-r^2/2\sigma r^2) dr + 2V \int_{-V}^V r \exp(-r^2/2\sigma r^2) dr \\
 &\quad + \int_V^{\infty} V^2 \exp(-r^2/2\sigma r^2) dr + \int_V^{\infty} r^2 \exp(-r^2/2\sigma r^2) dr \\
 &\quad \left. + 2V \int_V^{\infty} r \exp(-r^2/2\sigma r^2) dr \right] \\
 &= \frac{1}{\sigma r \sqrt{2\pi}} \left[4V \int_V^{\infty} r \exp(-r^2/2\sigma r^2) dr \right.
 \end{aligned}$$

$$+2V^2 \int_0^V \exp(-r^2/2\sigma r^2) dr + 2 \int_0^V r^2 \exp(-r^2/2\sigma r^2) dr] \quad (C.1)$$

The first two integrations in (C.1) are already evaluated in Appendix B and the third integration is evaluated as follows:

$$\begin{aligned} \int_0^V r^2 \exp(-r^2/2\sigma r^2) dr &= (r \int r \exp(-r^2/2\sigma r^2) dr) \Big|_0^V \\ &\quad - \int_0^V -\sigma r^2 \exp(-r^2/2\sigma r^2) dr \\ &= -\sigma r^2 V \exp(-V^2/2\sigma r^2) + \sigma r^2 \int_0^V \exp(-r^2/2\sigma r^2) dr \\ &= -\sigma r^2 V \exp(-V^2/2\sigma r^2) + \sigma r^2 \frac{\sigma r \sqrt{\pi}}{\sqrt{2}} \text{Erf}(V/\sqrt{2}\sigma r) \end{aligned}$$

Substitution of this result and results of Appendix B in Eq. (C.1) leads to

$$\begin{aligned} \langle (r+V) | r+V \rangle &= \frac{1}{\sigma r \sqrt{2\pi}} [4V\sigma r^2 \exp(-V^2/2\sigma r^2) + \\ &\quad + 2V^2 \frac{\sigma r \sqrt{\pi}}{\sqrt{2}} \text{Erf}(V/\sqrt{2}\sigma r) - 2V\sigma r^2 \exp(-V^2/2\sigma r^2) \\ &\quad + 2\sigma r^2 \frac{\sigma r \sqrt{\pi}}{\sqrt{2}} \text{Erf}(V/\sqrt{2}\sigma r)] \\ &= \frac{2V\sigma r}{\sqrt{2\pi}} \exp(-V^2/2\sigma r^2) + \left(\frac{2V^2}{\sqrt{2\pi}\sqrt{2}} + \frac{2\sigma r^2 \sqrt{\pi}}{\sqrt{2\pi}/2} \right) \text{Erf}(V/\sqrt{2}\sigma r) \\ &= \sqrt{\frac{2}{\pi}} V e^{-V^2/2\sigma r^2} + (V^2 + \sigma r^2) \text{Erf}(V/\sqrt{2}\sigma r) \end{aligned}$$

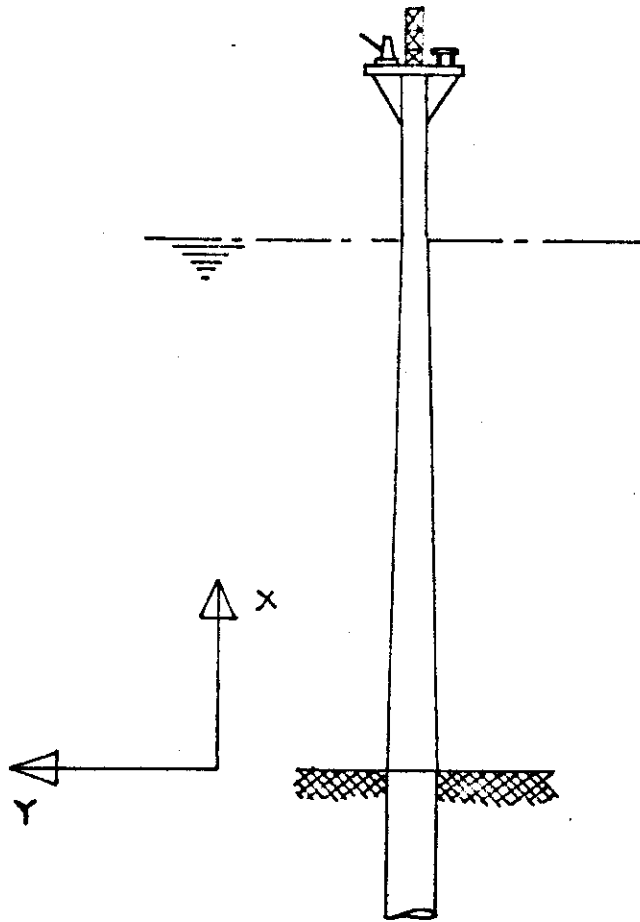


FIG.1 CAISSON

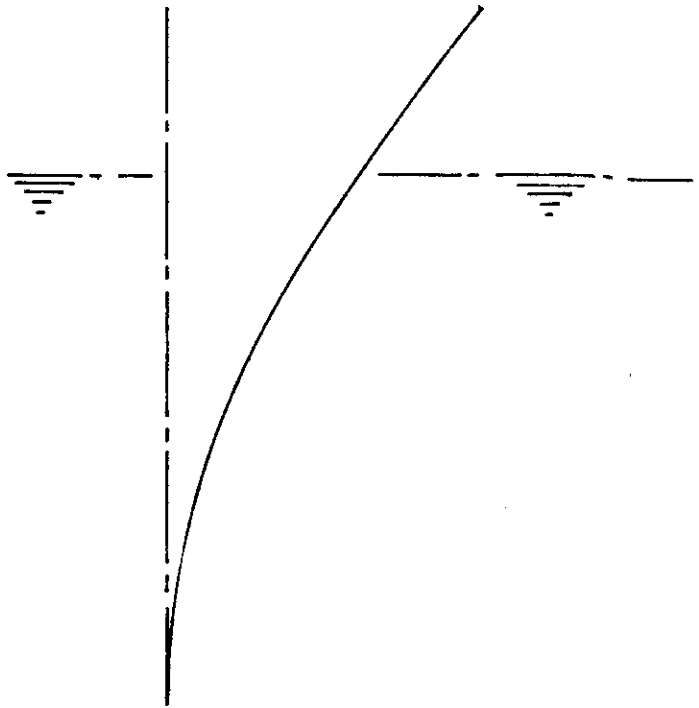


FIG.2 CAISSON MODE
SHAPE

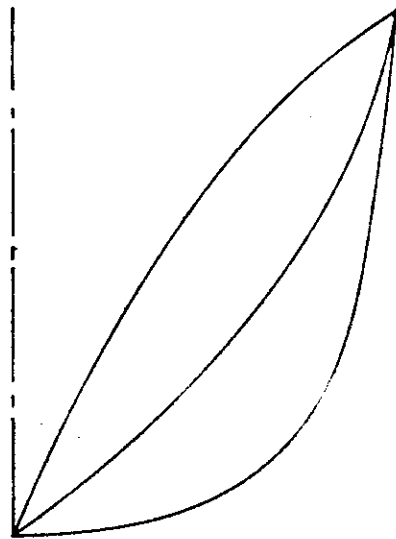


FIG.3 CURRENT DISTRIBUTION

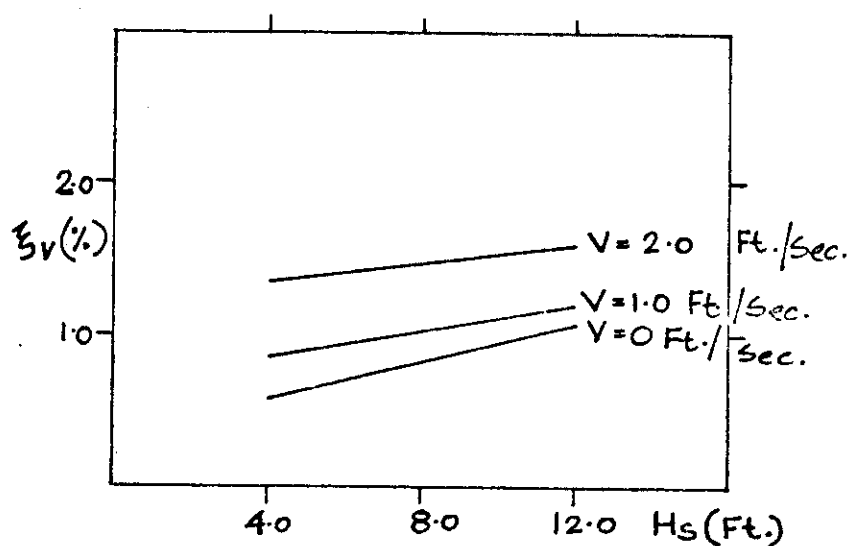


FIG. 4 SIGNIFICANT WAVE HEIGHT (H_s) VS.

NODAL VERTICAL DAMP RATE RATIO (ξ_v)

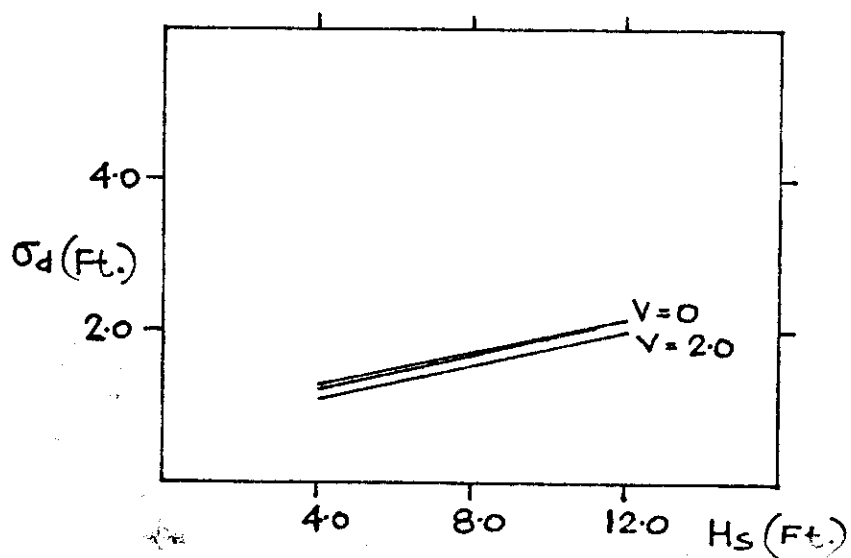


FIG. 5 DECK RESPONSE STD. DEVIATION (σ_d) VS.

SIGNIFICANT WAVE HEIGHT (H_s)

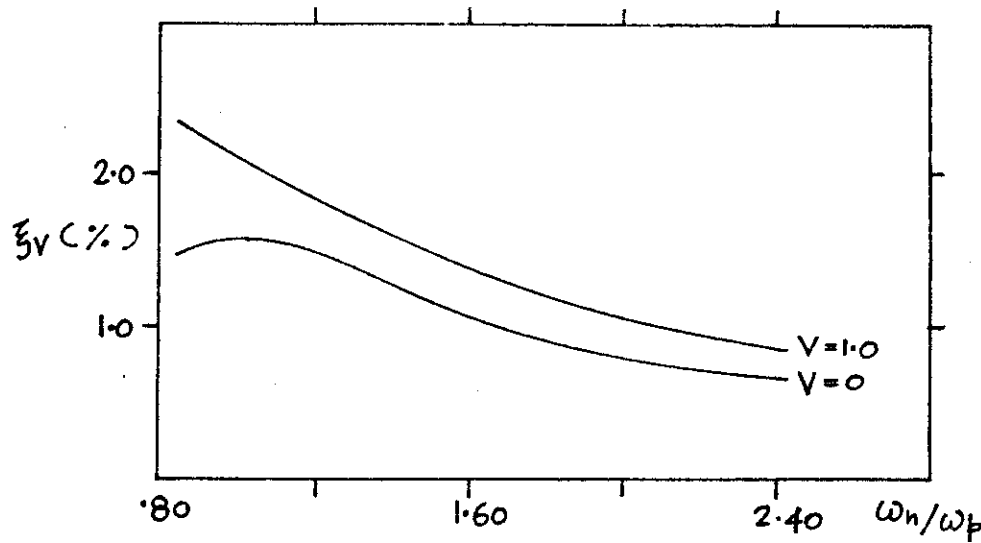


FIG. 6 MODAL VISCOUS DAMPING RATIO (ξ_v)
VS. FREQUENCY RATIO (ω_n/ω_p)

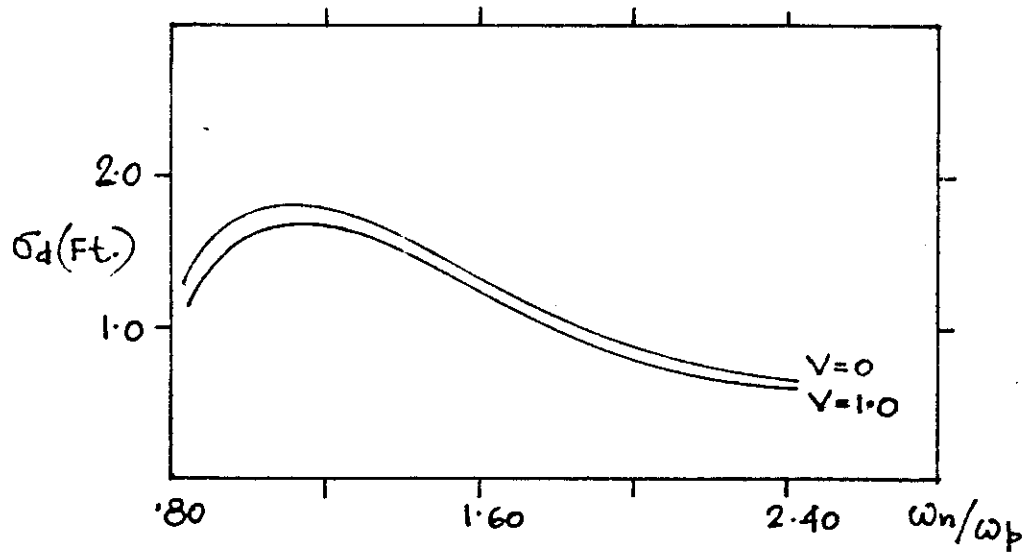


FIG. 7 DECK RESPONSE STD. DEVIATION (σ_d)
VS. FREQUENCY RATIO (ω_n/ω_p)

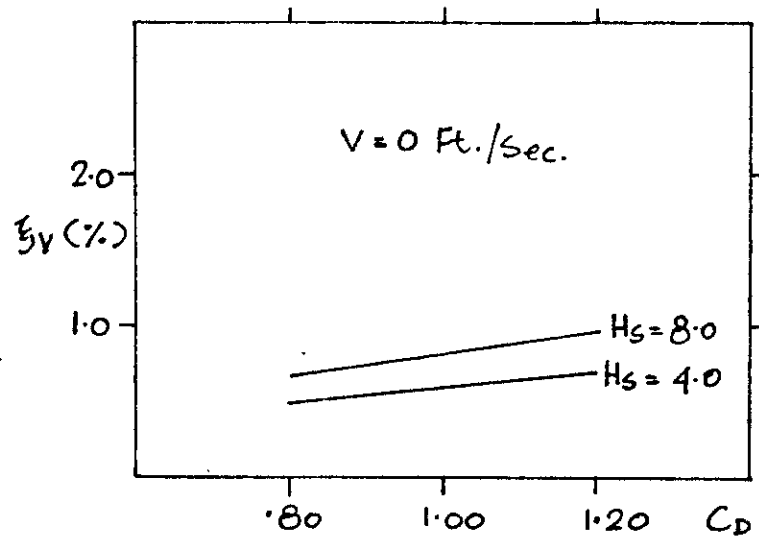


FIG. 8 MODAL VISCOUS DAMPING RATIO (ξ_V)
VS. DRAG COEFFICIENT (C_D)

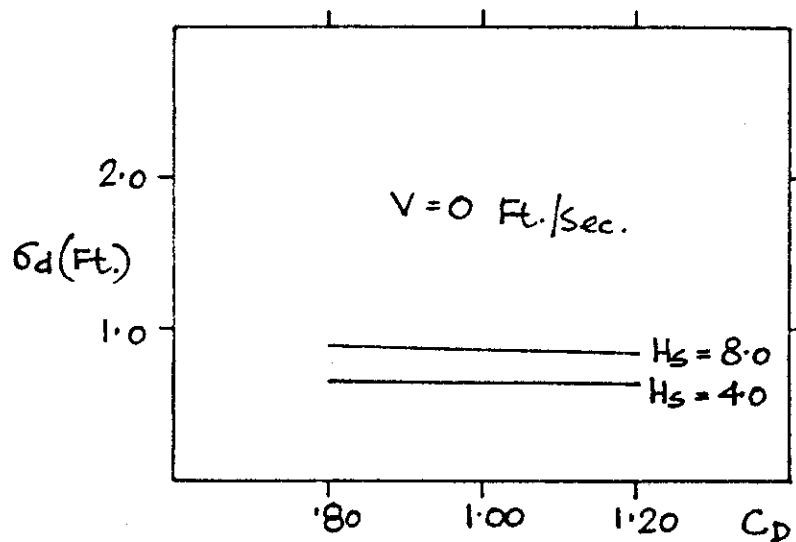


FIG. 9 DECK RESPONSE STD. DEVIATION (σ_d)
VS. DRAG COEFFICIENT (C_D)

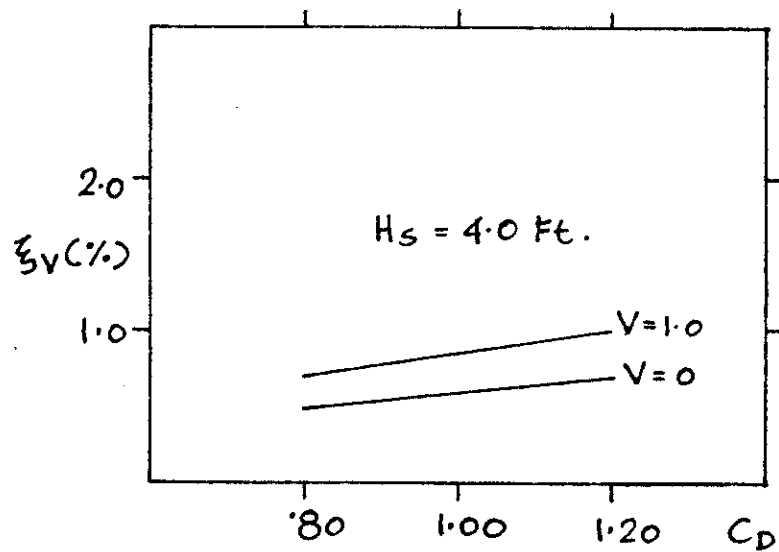


FIG. 10 MODAL VISCOUS DAMPING RATIO (ξ_V)
VS. DRAG COEFFICIENT (C_D)

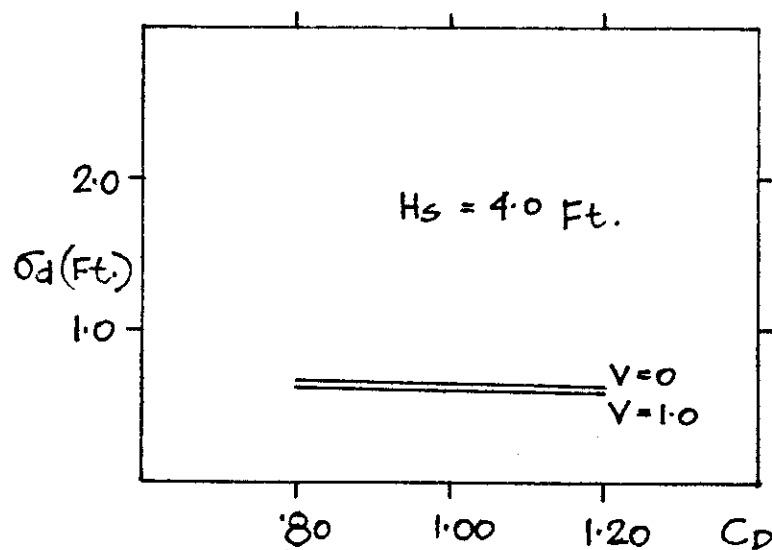


FIG. 11 DECK RESPONSE STD. DEVIATION (σ_d)
VS. DRAG COEFFICIENT (C_D)

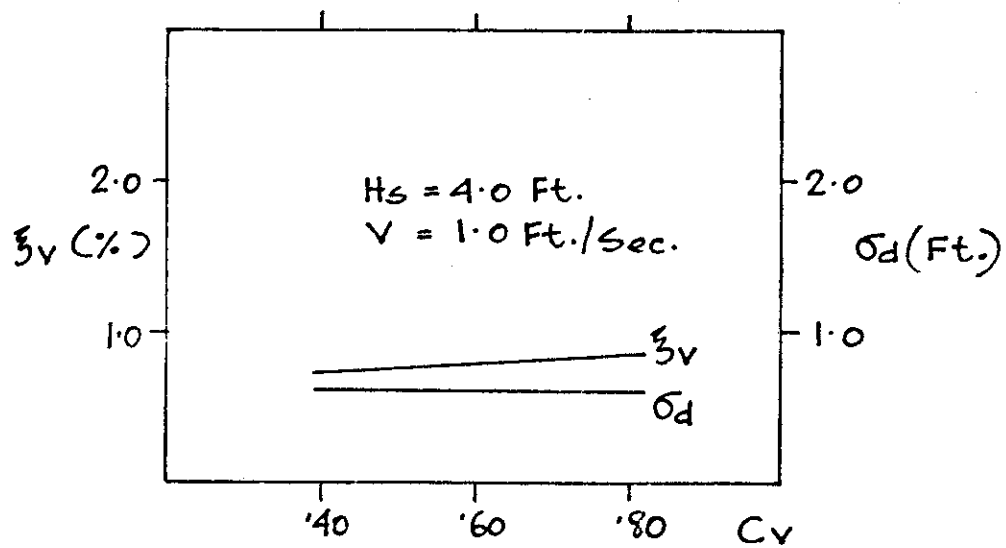


FIG. 12 MODAL VISCOUS DAMPING RATIO (ξ_v)
AND DECK RESPONSE STD. DEVIATION (σ_d)
VS. CURRENT DISTRIBUTION PARAMETER (C_v)

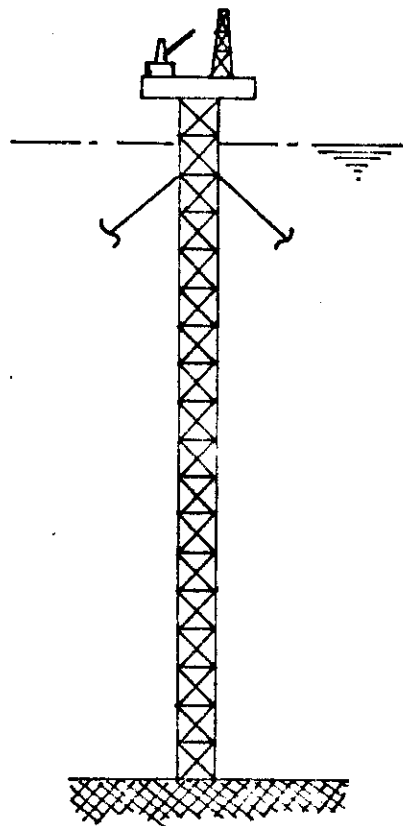


FIG. 13 GUYED TOWER

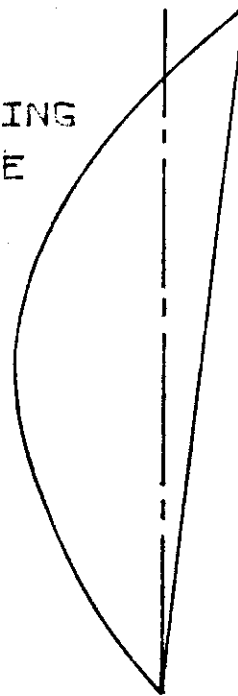
BENDING
MODESWAY
MODE

FIG. 14 TOWER MODE SHAPES

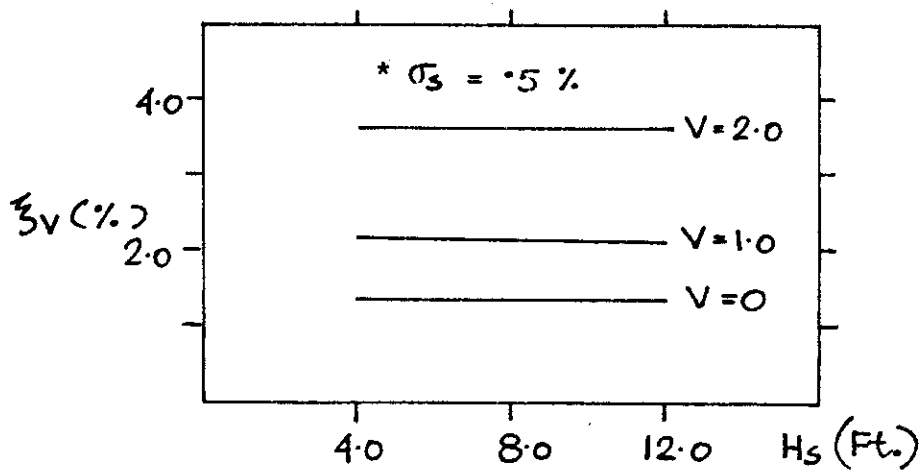


FIG. 15 MODAL VISCOUS DAMPING RATIO (ξ_v)
VS. SIGNIFICANT WAVE HEIGHT (H_s)

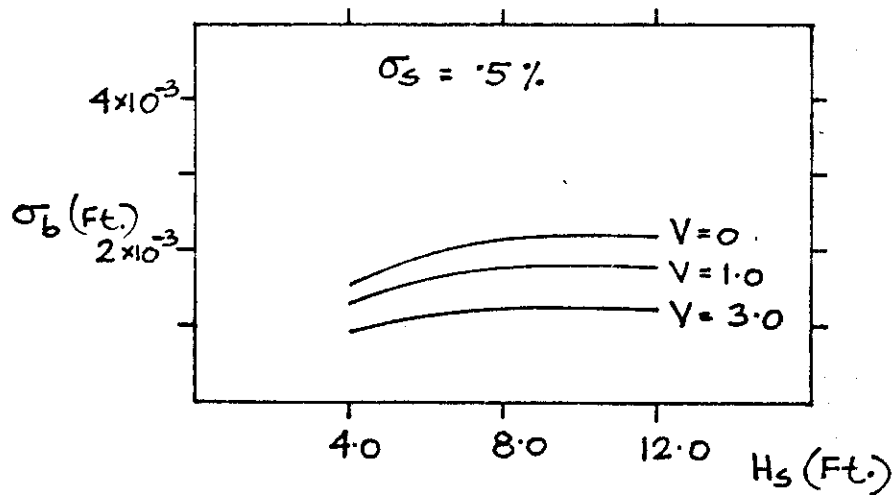


FIG. 16 DECK RESPONSE STD. DEVIATION (σ_b)
VS. SIGNIFICANT WAVE HEIGHT (H_s)

* σ_s is given as a percentage of tower length.

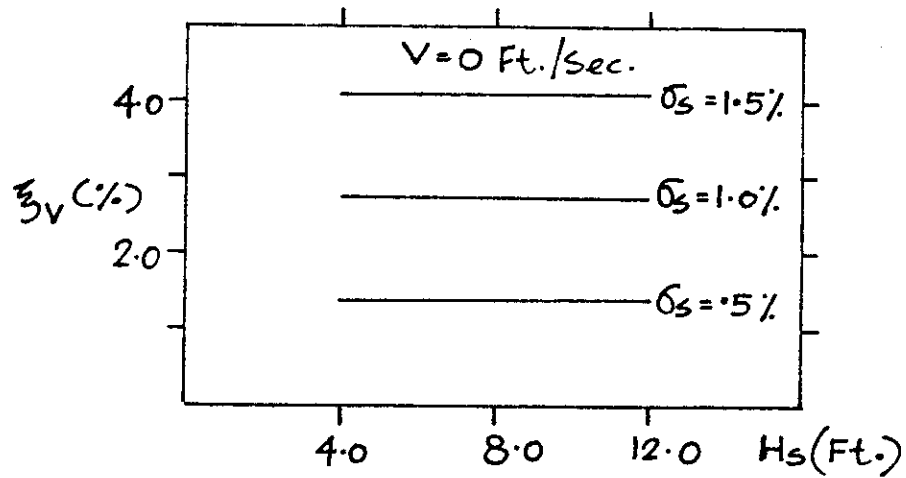


FIG. 17 MODAL VISCOUS DAMPING RATIO (ξ_v)
VS. SIGNIFICANT WAVE HEIGHT (H_s)

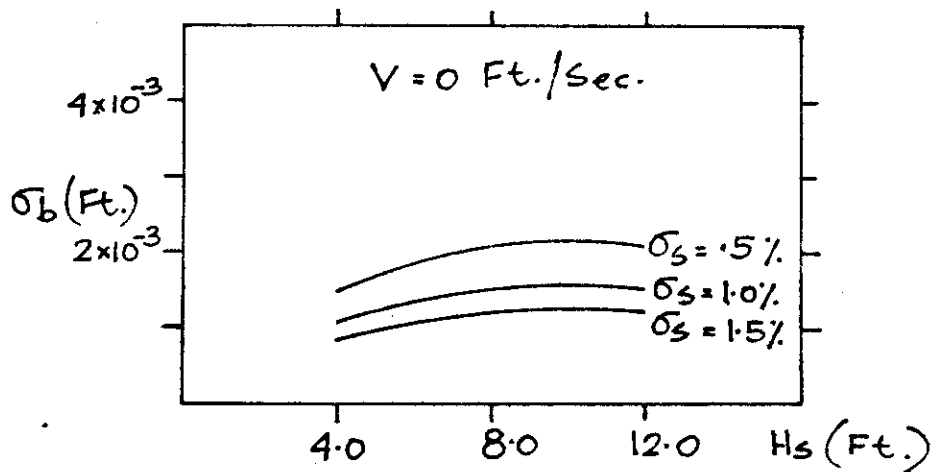


FIG. 18 DECK RESPONSE STD. DEVIATION (σ_b)
VS. SIGNIFICANT WAVE HEIGHT (H_s)

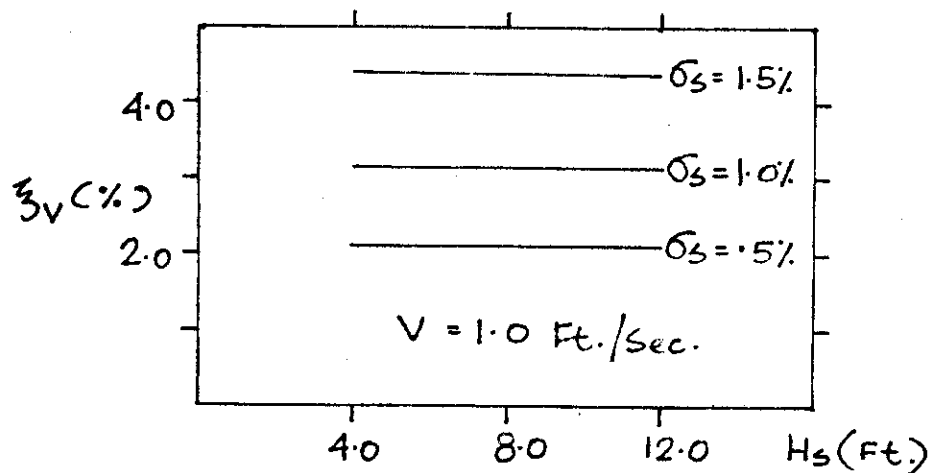


FIG. 13 MODAL VISCIOUS DAMPING RATIO (ξ_v)
VS. SIGNIFICANT WAVE HEIGHT (H_s)

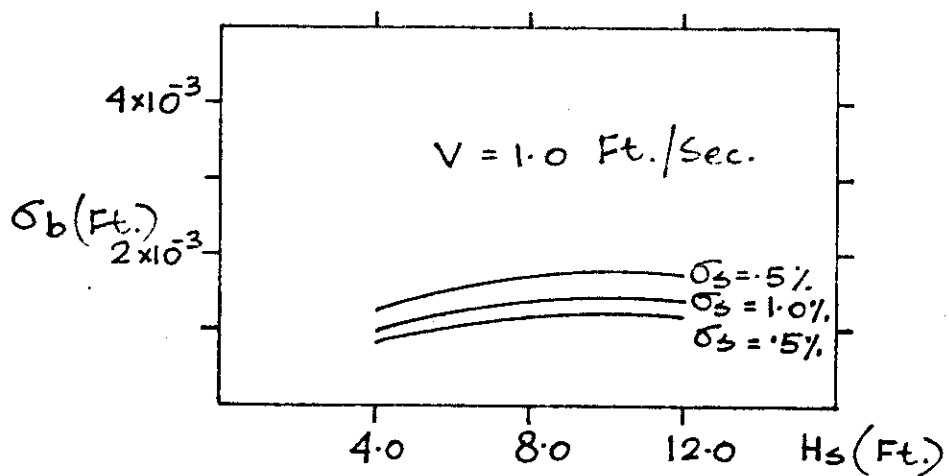


FIG. 20 DECK RESPONSE STD. DEVIATION (σ_b)
VS. SIGNIFICANT WAVE HEIGHT (H_s)

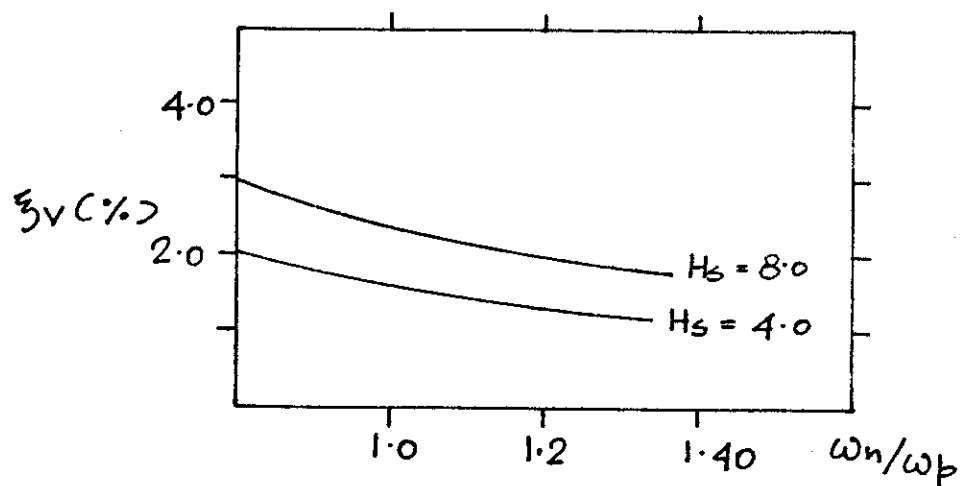


FIG. 21 MODAL VISCOUS DAMPING RATIO (ξ_v)
VS. FREQUENCY RATIO (ω_n/ω_p)

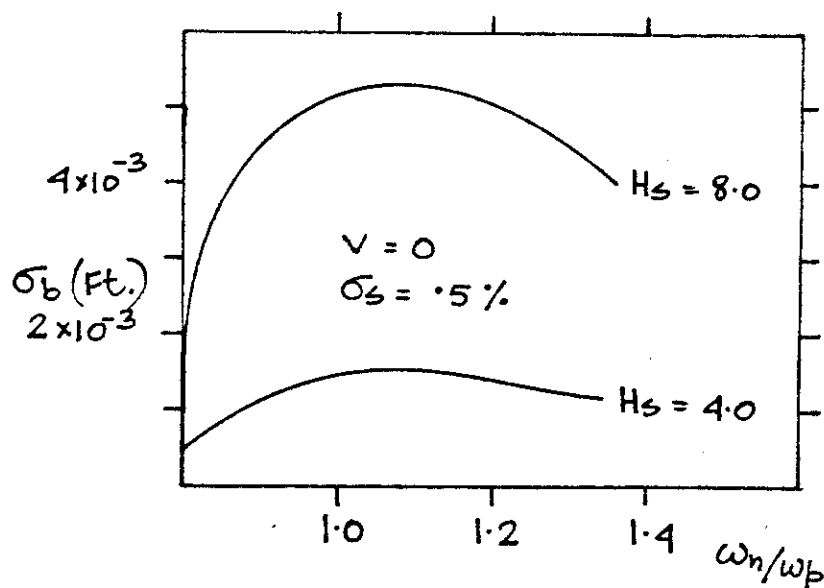
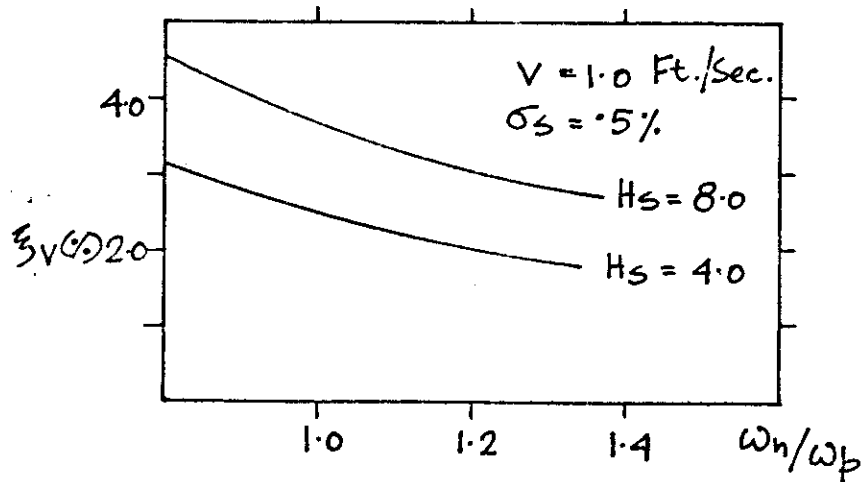
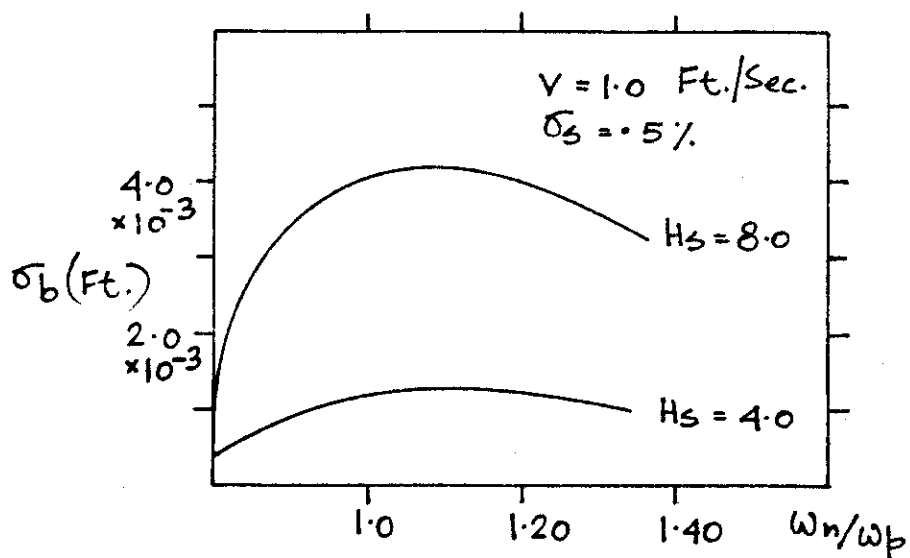


FIG. 22 DECK RESPONSE STD. DEVIATION (σ_b)
VS. FREQUENCY RATIO (ω_n/ω_p)

FIG. 23 MODAL VISCOUS DAMPING RATIO (ξ_v)VS. FREQUENCY RATIO (ω_n/ω_p)FIG. 24 DECK RESPONSE STD. DEVIATION (σ_b)VS. FREQUENCY RATIO (ω_n/ω_p)

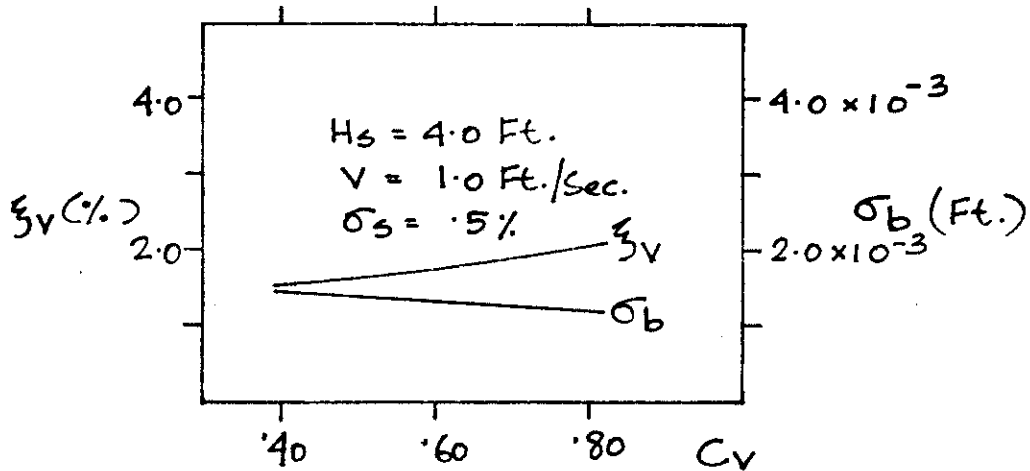


FIG. 25 MODAL VISCOUS DAMPING RATIO (ξ_v)
AND DECK RESPONSE STD. DEVIATION (σ_b)
VS. CURRENT DISTRIBUTION PARAMETER (C_v)

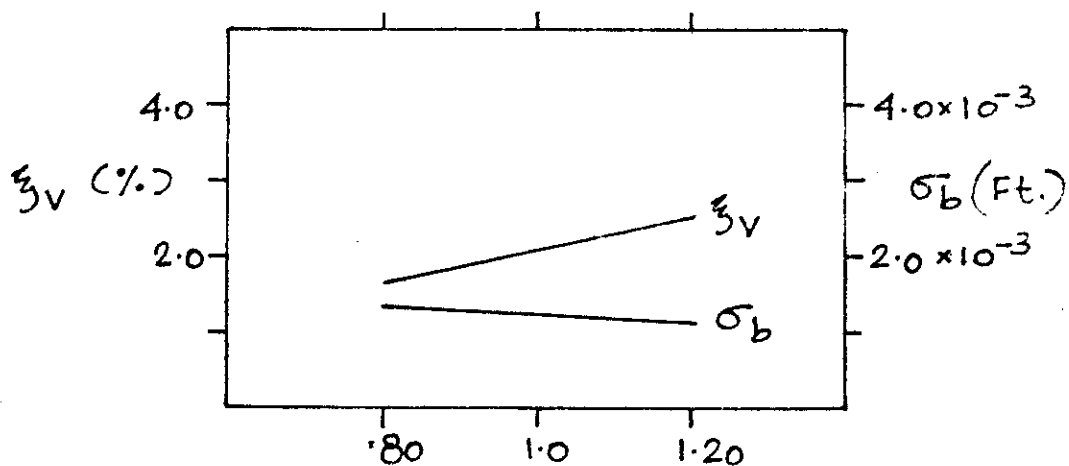


FIG. 26 MODAL VISCOUS DAMPING RATIO (ξ_v)
AND DECK RESPONSE STD. DEVIATION (σ_b)
VS. DRAG COEFFICIENT (C_d)